Combining logics using two-layer modal syntax

Petr Cintula

Institute of Computer Science
Academy of Sciences of the Czech Republic

Joint work with Carles Noguera

Two-layer modal syntax is based on two propositional languages (lower and upper one) and a modal language (a collection of modalities together with their arities) and features three kinds of formulae: (i) non-modal formulae build using *lower* propositional language, (ii) atomic modal formulae obtained by applying the modalities to *non-modal* ones, and (iii) complex modal formulae built from the atomic ones using the *upper* propositional language (i.e., the modalities cannot be nested and propositional languages cannot be mixed).

Early examples of logics employing this kind of syntax were modal logics of uncertainty inspired by Hamblin's seminal idea of reading the modal operator $P\varphi$ as 'probably φ' [6], meaning that the probability of φ is bigger than a given threshold (later elaborated by Fagin, Halpern and many others (see e.g. [2, 5]).

These initial examples used the classical logic to govern the behavior of formulae of both modal and non-modal layer. An interesting departure from the classical paradigm has been proposed by Hájek and Harmancová in [4] (later developed in Hájek's monograph [3]): they kept the classical logic as interpretation of the lower syntactical layer, but proposed to use Łukasiewicz logic in the upper layer, so that the truth degree of $P\varphi$ could be directly identified with the probability of φ. Later, other authors changed even the logic governing in the lower layer (e.g., to other fuzzy logic in order to allow for a treatment of uncertainty of vague events).

This research gave rise to an interesting way of combining logics which allows to use one logic to reason about formulae (rules) of other logic. The aim of this talk is to propose foundations for further research in this promising area. We define an abstract notion of a two-layer syntax and logic, a general semantics of *measured* Kripke frames and prove two forms of completeness theorem.

Acknowledgements: This research is supported by grant GAP202/10/1826 of the Czech Science Foundation. Petr Cintula also acknowledges the support of RVO 67985807; Carles Noguera was also supported by the FP7 PIRSES-GA-2009-247584 project MaToMUVI. This talk is based on [1].

References

[1] P. Cintula and C. Noguera. Modal logics of uncertainty with two-layer syntax: A general completeness theorem. In U. Kohlenbach, P. Barceló, and R. J. de Queiroz, editors, *Logic, Language, Information, and Computation - WoLLIC 2014*, volume 8652 of *Lecture Notes in Computer Science*, pages 124–136. Springer, 2014.

[2] R. Fagin, J. Y. Halpern, and N. Megiddo. A logic for reasoning about probabilities. *Information and Computation*, 87(1–2):78–128, 1990.

[3] P. Hájek. *Metamathematics of Fuzzy Logic*, volume 4 of *Trends in Logic*. Kluwer, Dordrecht, 1998.

[4] P. Hájek and D. Harmancová. Medical fuzzy expert systems and reasoning about beliefs. In M. S. Pedro Barahona and J. Wyatt, editors, *Artificial Intelligence in Medicine*, pages 403–404, Berlin, 1995. Springer.

[5] J. Y. Halpern. *Reasoning About Uncertainty*. MIT Press, 2005.

[6] C. L. Hamblin. The modal 'probably'. *Mind*, 68:234–240, 1959.