Baker-Beynon duality for Riesz MV-algebras

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The Baker-Beynon duality [1, 2] is a duality between lattice-ordered structures and suitable subspaces of $\mathbb{R}^n$, for some n.

We will focus our attention on the case of Riesz Spaces with strong unit, and consequently on Riesz MV-algebras. Therefore, we present work in progress on the Baker-Beynon duality for Riesz MV-algebras. We recall that a Riesz MV-algebra is an MV-algebra endowed with a scalar product with scalars from $[0, 1]$.
Denoted by **fgpRMV** the category of finitely generated and projective Riesz MV-algebras with homomorphisms of Riesz MV-algebras and $\mathcal{P}_{\mathbb{R}}$ the category of polyhedra with piecewise linear maps, our first result is the following.

Theorem 1. *The category* **fgpRMV** *is equivalent to the dual of the category* $\mathcal{P}_{\mathbb{R}}$*.*

Similar results hold for MV-algebras [5, 4], where the structures involved are finitely presented MV-algebras.
We connect finitely presented MV-algebras, finitely generated MV-algebras and projective MV-algebras to finitely presented Riesz MV-algebras, finitely generated Riesz MV-algebras and projective Riesz MV-algebras respectively. The main tool is the semisimple tensor product of MV-algebra: in [3] we have displayed an adjunction between semisimple MV-algebras and semisimple Riesz MV-algebras. From MV-algebras to Riesz MV-algebras we get the following.

Theorem 2. *i) Let A be a semisimple MV-algebra. If A is finitely generated, then $[0, 1] \otimes A$ is a finitely generated Riesz MV-algebra;*

ii) Let A be a semisimple MV-algebra. If A is projective, then $[0, 1] \otimes A$ is a projective Riesz MV-algebra;

iii) Let A be a semisimple MV-algebra. If A is finitely presented, then $[0, 1] \otimes A$ is a projective Riesz MV-algebra.

On the other side, if $\mathcal{U}_{\mathbb{R}}$ is the usual forgetful functor from Riesz MV-algebras to MV-algebras, we have the following.

Theorem 3. *i) Let A be a Riesz MV-algebra. If $\mathcal{U}_{\mathbb{R}}(A)$ is projective, then A is projective;*

ii) Let R be a Riesz MV-algebra. If $\mathcal{U}_{\mathbb{R}}(R)$ is finitely generated, then R is finitely generated.

Finally, in order to connect finitely presented structures, we deal with two special categories.

i) MV_{fp}^{pres} is the category whose objects are couple (X, I) with X non-empty set and I principal ideal in the free MV-algebra generated by X and usual homomorphism of MV-algebras;

ii) RMV_{fp}^{pres} is the category whose objects are couple (X, I) with X non-empty set and I principal ideal in the free Riesz MV-algebra generated by X and usual homomorphism of Riesz MV-algebras.

We prove display an equivalence between MV_{fp}^{pres} and RMV_{fp}^{pres}. Moreover, any object of MV_{fp}^{pres} (RMV_{fp}^{pres} respectively) is in one-one correspondence with a suitable finitely presented MV-algebra (Riesz MV-algebra) and any morphism in MV_{fp}^{pres} or RMV_{fp}^{pres} induce a morphism between finitely presented MV-algebras or Riesz MV-algebras.

References

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