# From Freudental's Spectral Theorem to projectable hulls of unital Archimedean lattice-groups, through compactification of minimal spectra part 2: the general case.

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This is the second part of a series of two presentations, the first being by Andrea Pedrini.

In its basic version, Freudenthal's Spectral Theorem [4] asserts that any element of a Riesz space $R$ with a strong unit $u$ and the principal projection property may be uniformly approximated, in the norm that $u$ induces on $R$, by abstract characteristic functions – "components of the unit $u$". Freudenthal's theorem led to a considerable amount of research on Riesz spaces and their generalisations, the lattice-ordered Abelian groups that concern us here. (See [8] and [3, 6] for background.) One main line of research concentrated on extending one given structure $G$ to a minimal completion that enjoys the principal projection property, where Freudenthal's theorem therefore applies. Such an extension is called *the projectable hull* of $G$. Here we present a new construction of the projectable hull of an Archimedean $\ell$-group equipped with a strong order unit $u$ that does not use direct limits, nor essential closures. Our construction exposes instead the intimate connection between projectable hulls and zero-dimensional compactifications of spectral spaces of minimal prime ideals.

In this second talk, we quickly review some of the basic definitions concerning the notions involved in the construction and particularly consider when the space $\operatorname{Min} G$ — a Hausdorff zero-dimensional space — is not compact [1]. For our construction in this case, we must recall the definition of a Wallman base for a topology as well as a Wallman compactification [9].

Here, the base $\{\mathbb{V}_m(g)\}_{g\in G}$ for the closed sets of $\operatorname{Min} G$ is *n*ot Boolean algebra under set-theoretic union, intersection and complementation and does *n*ot coincide with the set of all clopen subsets of $\operatorname{Min} G$ [2, 10].

However, we note that $\{\mathbb{V}_m(g)\}_{g\in G}$ does form a Wallman base for $\operatorname{Min} G$, consisting of clopen sets, from which we may construct a special zero-dimensional Wallman compactification of $\operatorname{Min} G$ which we denote $\beta_0^G(\operatorname{Min} G)$. Similar to the previous talk, we now have an embedding of $G$ into $C(\beta_0^G(\operatorname{Min} G))$ and letting $\widetilde{G}$ be the image of $G$ and $K$ collection of continuous characteristic functions on $\beta_0^G(\operatorname{Min} G)$, we may construct the projectable hull of $G$, $\mathcal{P}(G)$, as the $\ell$-subgroup of $\mathrm{C}(\beta_0^G(\operatorname{Min} G))$ generated by $\widetilde{G} \cup K$. We note that the compactification $\beta_0^G(\operatorname{Min} G)$ is specifically constructed to have only those continuous characteristic functions derived from $G$ and that $\operatorname{Max} \mathcal{P}(G) \cong \beta_0^G(\operatorname{Min} G)$.

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