

# Hoops and Intuitionistic Łukasiewicz Logic

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We study multi-valued logic of Łukasiewicz LL as an extension of intuitionistic affine logic IAL. Two extra axioms need to be added to IAL to obtain LL: the classical axiom of double negation elimination DNE, and the intuitionistically valid axiom of divisibility AD. The intermediate logic IAL + AD we call intuitionistically Łukasiewicz logic ILL. Hoops are the algebraic structures that provide natural models for IAL + AD. The logic ILL turns out to be surprisingly powerful fragments of intuitionistic logic, as we demonstrate by presenting:

- a simpler proof of the Ferreirim-Veroff-Spinks theorem;
- a proof that idempotent elements of a hoop form a sub-hoop;
- a proof that double negation is a hoop homomorphism; and
- proofs for intuitionistically valid versions of all the De Morgan dualities.

However most of the proofs are very intricate and have been found with computer assistance. We have factored these proofs (also with some computer assistance) into what we believe is a natural and understandable sequence of lemmas and theorems. The presentation is further simplified by the introduction of four derived connectives, which also satisfy natural De Morgan dualities. We conclude by showing that the homomorphism property of the double-negation mapping implies that all the standard negative translations of classical into intuitionistic Łukasiewicz coincide, as they do in full intuitionistic logic. This is in contrast with affine logic for which we will see that both Gentzen and Glivenko translations fail.