Lattice-ordered abelian groups and perfect MV-algebras: a topos-theoretic perspective

Anna Carla Russo

University of Salerno

This talk is based on [2]. We establish, generalizing Di Nola and Lettieri's categorical equivalence [3], a Morita-equivalence between the theory of lattice-ordered abelian groups and that of perfect MV-algebras. Further, after observing that the two theories are not bi-interpretable in the classical sense, we identify, by considering appropriate topos-theoretic invariants on their common classifying topos according to the 'bridge technique' of [1], three levels of bi-interpretability holding for particular classes of formulas: irreducible formulas, geometric sentences and imaginaries. Lastly, by investigating the classifying topos of the theory of perfect MV-algebras, we obtain various results on its syntax and semantics also in relation to the cartesian theory of the variety generated by Chang's MV-algebra, including a concrete representation for the finitely presentable models of the latter theory as finite products of finitely presentable perfect MV-algebras. Among the results established on the way, we mention a Morita-equivalence between the theory of lattice-ordered abelian groups and that of cancellative lattice-ordered abelian monoids with bottom element.

References

[1] O. Caramello, The unification of Mathematics via Topos Theory, *arXiv:math.CT/1006.3930* (2010).

[2] O. Caramello and A.C. Russo, Lattice-ordered abelian groups and perfect MV-algebras: a topos-theoretic perspective, *arXiv:math.CT/1409.4730* (2014).

[3] A. Di Nola and A. Lettieri, Perfect MV-Algebras are Categorically Equivalent to Abelian l-Groups, *Studia Logica* 88 (1958), 467-490.